

LBL-15622
EEB-W 83-04
W-139

Presented at the International Daylighting Conference, Phoenix AZ, February 16-18, 1983, and to be published in *Energy and Buildings*.

ZENITH LUMINANCE AND SKY LUMINANCE DISTRIBUTIONS FOR DAYLIGHTING CALCULATIONS

M. Karayel, M. Navvab, E. Ne'eman, and S. Selkowitz

Energy Efficient Buildings Program
Lawrence Berkeley Laboratory
University of California
Berkeley CA 94720 USA

May 1983

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Building Energy Research and Development, Building Systems Division, of the U.S. Department of Energy under Contract No. DE-ACO3-76SF00098.

ZENITH LUMINANCE AND SKY LUMINANCE DISTRIBUTIONS FOR DAYLIGHTING CALCULATIONS

M. Karayel, M. Navvab, E. Ne'eman, and S. Selkowitz

Energy Efficient Buildings Program
Lawrence Berkeley Laboratory
University of California
Berkeley CA 94720 USA

ABSTRACT

We derive an equation for zenith luminance as a function of turbidity and solar altitude based on analysis of large quantities of luminance and illuminance data measured in San Francisco, California, between September 1979 and August 1982. Using only average turbidity values to predict hourly zenith luminance as a function of solar altitude can produce large errors. We compare the equation derived from our data, which is valid over a wide range of turbidity and solar altitude, to other published models. We also compare the relationship between horizontal illuminance and zenith luminance from the clear sky and conclude that, when ideal clear days are compared, this relationship is similar to earlier work based on measurements in European climates. Finally, we compare our sky luminance distribution measurements to previous published luminance distributions using the diffusion indicatrix. Our results are intended to help improve daylight availability prediction techniques and define additional requirements for data collection.

INTRODUCTION

Daylight availability studies are motivated by the realization that sky conditions differ in different localities and that data are often gathered on only a few solar and climatological variables. As a result, statistical methods are often used to derive key parameters. Extrapolation from limited measured data seems to be a common practice.

Zenith luminance is integral to many daylight availability calculations. We present results of a study of zenith luminance in which systematic automatic and manual measurements were made for three years in San Francisco, California. We compare our results with previous work on this subject and examine the validity of extrapolating results from one locality to others. The collected data are analyzed statistically to determine parameters and functional relationships.

Because our data acquisition is automatic, we had to distinguish sky conditions based on criteria derived solely from solar illuminance and irradiance measurements, rather than subjective human judgment. We generally used the criterion that the sky is clear as long as the direct normal irradiance is greater than 200 watts/m^2 . This criterion can be met when there are thin layers of high clouds; to exclude these conditions (when solar altitude is higher than 20°), we also require that the ratio of diffuse irradiance to global irradiance be less than $1/3$. In this paper we analyze only the clear sky data that satisfy these criteria. Note that the analyzed data may include data from partly cloudy days when the sun was shining.

DATA COLLECTION

The data-gathering station is located on the roof of the Pacific Gas and Electric (PG&E) building in downtown San Francisco, approximately 140 m (450 ft) above sea level. Measurements of eight irradiance and illuminance variables have been taken there every 15 minutes for more than four years. At the same location, measurements of zenith luminance have been taken every 15 minutes for more than three years. The station instrumentation consists of seven illuminance sensors (photometers) and two irradiance sensors (pyranometers) that measure global and diffuse components. For both diffuse illuminance and diffuse irradiance measurements, the sun is blocked by a shadow band. Four of the photometers measure vertical illuminances for each major compass orientation. In order to eliminate ground reflectance, a circular black honeycomb plane with a metal ring around its edge is mounted below the vertical sensors.

Zenith luminance is measured by a modified photometer. A cylindrical steradian shade assembly is mounted on this sensor, permitting a 15° field of view of the zenith region (see Fig. 1). For clear conditions, the results are corrected to the standard 1° field of view used for zenith luminance measurements. This is done by integrating the sky luminance distribution for a standard CIE clear sky for both of these angles and comparing the results. The area-corrected error was less than 1% for solar altitudes less than 80° . Because the maximum solar altitude at our location is about 76° , we believe these results are acceptable.

Additional manual measurements were made of the sky luminance distribution with a 1° luminance sensor and a camera fitted with a fisheye orthographic lens. These measurements were carried out in two locations: on top of the PG&E building and at the Environmental Design building on the University of California campus in Berkeley, across the San Francisco Bay. The two locations are about 20 kilometers apart. These manual measurements were made to obtain typical sky luminance distributions and to test whether the CIE clear sky

models are appropriate for sky luminance patterns in this region.

To describe our data fully, we must mention San Francisco's unique microclimate. The city is located at 38° north latitude and 123° longitude on a peninsula between the Pacific Ocean to the west and a large bay to the east. San Francisco is a naturally air-conditioned city with cool, pleasant summers and mild winters due to the interplay of westerly air masses and cool ocean currents from the north. Some of the most unusual aspects of this city's microclimate are the sea fog and low clouds in early mornings and the variability of clouds within different parts of the city. Our automatically gathered data include representative samples of all the sky conditions common to San Francisco.

RESULTS

Zenith luminance equation

Zenith luminance is usually expressed as a function of solar altitude, γ_s , and Linke's turbidity factor, T_L . Several authors have developed different analytical expressions for clear sky zenith luminance and for Linke's turbidity factor. We calculated the turbidity factor directly from horizontal irradiance and illuminance measurements. Details of these calculations are presented in the authors' paper on turbidity [1].

Because we took measurements over a wide range of turbidity and for higher solar altitudes than have most other investigators, their equations proved inadequate to fit our data. Table 1 lists some proposed equations and their limitations. To fit our data, we simplified the general form used by Dogniaux and Liebelt by omitting the linear dependence on turbidity and refit the parameters:

$$L_z = (1.376T_L - 1.81)\tan\gamma_s + 0.38, \quad (1)$$

where:

L_z = zenith luminance (kcd/m^2),

γ_s = solar altitude (degrees), and

T_L = Linke's turbidity factor.

The parameters of Eq. (1) were estimated from a set of 11,900 measurements by nonlinear least squares. The standard deviation for this fit was 0.637 kcd/m^2 . This equation is valid for $2^\circ \leq \gamma_s \leq 75^\circ$. Figure 2 shows the dependence of zenith luminance on turbidity and solar altitude based on Eq. (1). We derived other, more complicated equations to predict zenith luminance. But in general the small improvement in accuracy gained by using these equations did not justify the increased computational complexity.

Figures 3 and 4 show comparisons of our results with equations derived by several other authors for a high ($T_L = 5.5$) and a low ($T_L = 2.75$) turbidity value, respectively. The other authors whose equations we used for comparison generally give limited ranges of turbidity for which their equations are valid. We have insufficient information concerning the sources of data from which the other equations were derived to speculate on the reason for the differences shown in these figures, although the clear sky criterion is probably one factor.

Equation (1) predicts the general trend of zenith luminances. There may still be considerable scatter, however, between this equation and any specific set of measured data. Figures 5A through 5C displays the actual measurements and predicted zenith luminance from three randomly selected consecutive days in August 1980. Hours with missing data did not meet the

clear sky criteria. The agreement between our predictions and the measured data is quite good for this limited random sample.

We tested how well the equations in Table 1 fit our measured data for a wide range of turbidity. Statistical goodness of fit tests indicated that our equation is an unbiased estimator of the zenith luminance values. On the average, the equation given by Dogniaux overestimated our data by 1.024 kcd/m^2 , and that given by Liebelt underestimated our data by 0.679 kcd/m^2 over the range for which each equation is valid. Furthermore, even when this bias was removed our equation predicted zenith luminance with less deviation.¹ This is not surprising, since our equation was derived from this data set. We expect to test our predictive equation at other locations to determine if it provides a good fit to data elsewhere.

Since turbidity is measured in few locations, it is often necessary to estimate zenith luminance values based on only limited turbidity data. In order to examine the variability in zenith luminance data, we averaged the data over turbidities for each solar altitude. The standard deviation was also computed. Figure 6 shows this average value plus or minus one and two standard deviations, $(\sigma_{L_z}(\gamma_s))$. This figure shows that for a given solar altitude, zenith luminance can vary considerably from the calculated average value. With low altitudes, the relative difference is fairly high, but the absolute values remain low. This is not the case with altitudes above 60° , where the relative difference as well as the absolute values are both quite high. In a location for which turbidity data are not available, a calculation of zenith luminance using only the dependence on solar altitude at an average turbidity value can thus introduce a potentially large relative error.

Zenith Luminance and Diffuse Illuminance

Several authors have suggested formulae for the relationship between zenith luminance and horizontal illuminance from the sky for clear sky conditions [2]. Measuring these variables simultaneously makes it possible to examine this empirical relationship. Our data indicate that the ratio of horizontal illuminance to zenith luminance for clear sky first rises and then drops with solar altitude, as expected. More significantly, our average data suggest values for the relative zenith luminance, R_L (i.e., the ratio of diffuse horizontal illuminance to zenith luminance), of the clear sky that are lower than those derived from both the CIE sky luminance distribution according to Kittler and according to Gusev as reported by Aydinli [2]. We derived the following equation for the average value of the relative zenith luminance:

$$R_L = 5.42 - 4.16\gamma_s + 10.87\gamma_s e^{-2.2\gamma_s}. \quad (2)$$

(γ_s is given in radians). For a comparison of our fitted equation to that of Kittler or Gusev, see Fig. 7.

R_L can also be derived from clear sky luminance distribution data. We measured clear sky luminance distributions for San Francisco for a limited number of very clear days using manual survey techniques (see next section). R_L estimated by integrating our measured luminance distributions closely matches that calculated from the CIE distribution for all solar altitudes above 20° , but is lower for very low solar altitudes (see “ideal clear sky” curve in Fig. 7). This analysis suggests that the difference between our average equation and others’ may be

¹Standard deviation based on our equation was 0.798 kcd/m^2 , whereas the standard deviation according to the equations by Dogniaux and Liebelt were 1.078 and 1.041 kcd/m^2 , respectively.

attributed partly to the fact that our cumulative data set represents “average” rather than “ideal” clear sky conditions.

Aydinli suggests a seven-term power-series expansion for the relative zenith luminance: $R_L = \sum_{i=0}^6 A_i \gamma_s^i$ [2]. The power-series coefficients (A_i) are given in Table 2. Because power-series equations of this type do not increase one’s understanding of the empirical relationships, we prefer to display average measured data with confidence limits (Fig. 8). This figure shows the ratio of diffuse horizontal illuminance to zenith luminance as measured, and displays the scatter as ± 1 standard deviation ($\pm \sigma_{R_L}(\gamma_s)$).

Sky luminance distribution

The sky luminance distribution, including that of the zenith and circumsolar regions, was examined through manual surveys of the sky on clear days. For these measurements a visual evaluation was used as the criterion for a clear sky. The measurements were taken for every 10° azimuth starting at the sun azimuth and for every 10° of altitude. The sky was assumed to be symmetric with respect to the sun azimuth; therefore, only half of the sky was surveyed, so the survey was completed in less than 10 minutes, avoiding significant changes in sun position. Figure 9 shows a sample plot of clear sky luminance distribution (sun altitude = 60°) derived from analysis of our measured data.

In order to compare our measurements of the sky luminance distribution to others’, we have used the concept of a diffusion indicatrix developed by Kittler [4]. The diffusion indicatrix models the dependence of sky luminance distribution on atmospheric scattering phenomena. Equation (3) shows the diffusion indicatrix:

$$F(\theta, \gamma) = P_1(1 - e^{-P_4 \sec(\theta)})(1 + P_2(e^{-3\gamma} - 0.009) + P_3 \cos^2 \gamma), \quad (3)$$

where:

- P_1, P_2, P_3 , and P_4 are parameters estimated from measured data,
- θ = angle between the zenith and the sky element P, and
- γ = scattering angle between the sky element and the sun (see Fig. 10)

The scattering angle γ can be calculated using the equation:

$$\gamma = \cos^{-1}(\sin \theta \sin \theta_s \cos \alpha + \cos \theta \cos \theta_s), \quad (4)$$

where

- θ = angle between the sky element and the zenith,
- θ_s = angle between the sun and the zenith, and
- α = azimuth angle between the sun azimuth and the sky element azimuth.

The dashed circular line in Fig. 10 shows the locus of points in the sky which have the same γ value.

Based on Eq. (3) above, we estimated the parameters P_1 through P_4 for 172 data points from one clear day for each month of the year. The multiple correlation coefficients for these fits were high, indicating good fits to data. It was found, however, that the values of the parameters P_1, P_2, P_3 , and P_4 derived from analysis of our measured data had considerable scatter around the values given by Kittler and by Gusev in Ref. [3]. Table 3 compares the values of

the P coefficients suggested by Kittler and Gusev to those derived from best fits of our data to the diffusion indicatrix equation.

We compared results of our measurements to Kittler's and Gusev's equations using several different approaches. In Figs. 11a and 11b we plot values of the indicatrix derived from our measured data against Kittler's and Gusev's equations for the indicatrix. All values have been normalized for a scattering angle of $\gamma = 90^\circ$ by dividing each measured and predicted value by the value of the predicted diffusion indicatrix at $\gamma = 90^\circ$. The agreement is generally good for $\gamma < 45^\circ$, but is poorer for larger values of γ . We note that the average P_3 term calculated for our data has an opposite sign compared to Kittler and Gusev (see Table 3), but our P_2 value is larger than Kittler's. In addition, our average value for P_4 differs by 25%, and the indicatrix is particularly sensitive to the values of P_4 for the large values of θ that correspond to large values of γ . If we plot our data with our values for P_1 , P_4 and normalize to our indicatrix value at $\gamma = 90^\circ$, we obtain an excellent fit (Fig. 11c). The fit is unbiased in that the sign of the errors is randomly scattered with respect to scattering angle. This suggests that the form of the equation is suitable for analyzing our data. However, because atmospheric turbidity affects the shape of the diffusion indicatrix, different parameters ($P_1 - P_4$) may be appropriate for different locations. More detailed measurements and analysis to relate the four coefficients to atmospheric properties will be necessary to understand the basis for the differences between the values for the P values derived from our measured data and Kittler's and Gusev's values.

CONCLUSIONS

Equations for zenith luminance of the clear sky proposed by Gusev, Kittler, Dogniaux, Liebelt, and Krochmann are all limited to solar altitudes below 65° and the range of turbidities given in Table 1. Based on an extensive measured data set, we have derived a new zenith luminance equation that extends zenith luminance predictions to solar altitudes up to 76° and turbidities of 1.5 to 8.5. Given the diversity and quantity of our data, we believe that this equation is valid for a range of climatic conditions.

We derived an equation for relative zenith luminance (i.e., ratio of horizontal illuminance to zenith luminance) based on our measured data. Our data suggest lower values than those based on standard CIE distributions. We believe that the differences occur partly because our extensive data set represents "average" rather than ideal clear sky conditions. Further work is needed to clarify these differences.

We have also analyzed limited sky luminance distribution measurements and evaluated a diffusion indicatrix that describes this distribution as a function of the sun-to-sky-element angle and the zenith-to-sky-element angle. The illuminance levels on a given surface may be predicted by appropriately integrating this equation. The parameters of this equation, as given by the CIE [3], are strongly dependent on instantaneous atmospheric conditions. These parameters, as shown in Table 3, vary over a fairly wide range. However, the estimated average value of each parameter is in general agreement with the corresponding values proposed by Gusev and Kittler. Additional studies are required to determine direct relationships between turbidity and the parameters of the diffusion indicatrix.

ACKNOWLEDGEMENTS

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Building Energy Research and Development, Building Systems Division of the U.S. Department of Energy under Contract No. DE-ACO3-76SF00098.

Special thanks are due to Robert Clear of the Lighting Systems Research Group at Lawrence Berkeley Laboratory for his many helpful suggestions, to Moya Melody for her editorial assistance, and to Ruth Williams for word-processing.

REFERENCES

- [1] Navvab, M., Karayel, M., Ne'eman E., and Selkowitz, S., Measurement and Calculation of Atmospheric Turbidity, to be published in *Energy and Buildings*, Lawrence Berkeley Laboratory Report LBL 17727.
- [2] Aydinli, S., The Availability of Solar Radiation and Daylight, 2nd Draft of Daylighting Report for CIE Technical Committee TC-4.2, January 1983.
- [3] CIE Technical Committee 4.2, Standardization of Luminance Distribution of Clear Skies, CIE publication No. 22, 1973.
- [4] Kittler, R., and Kittlerova, L., *Navrh a Hodnotenie Denneho Osvetlenia*, Vydavatel'stvo Technickej a Ekonomickej Literatury, Bratislava, 1968, 1975.
- [5] CIE Technical Committee 4.2, Circular Letter No. 3/82, CIE, March 1982.
- [6] Bevington, P.R., *Data Reduction and Error Analysis for Physical Sciences*, McGraw-Hill Book Co., 1969.
- [7] Liebelt, C., *Leuchtdichte und Strahldichtererteilung des Himmels*, Dissertation, Lichttechnische Institut der Uni, Karlsruhe, 1978.

Table 1. Equations for Zenith Luminance†

In these equations:

L_Z = zenith luminance (kcd/m^2),

γ_s = solar altitude (degrees), and

T_L = Linke's turbidity factor.

Author	Equation	Limits	γ_s
Dogniaux	$L_Z = (1.234T_L - 0.252) \tan \gamma_s + 0.112T_L - 0.0169$		$\gamma_s \leq 65^\circ$
Kittler	$L_Z = 0.3 + 3.0 \tan \gamma_s$	$2.25 \leq T_L \leq 3.25$	$\gamma_s \leq 65^\circ$
Krochmann	$L_Z = 0.1 + 0.063 \gamma_s + (\gamma_s (\gamma_s - 30)/1000) e^{0.0346(\gamma_s - 68)}$	$T_L = 2.75$	
Liebelt	$L_Z = (1.34T_L - 3.46) \tan \gamma_s + 0.1T_L + 0.9$	$3 \leq T_L \leq 7.5$	
Gusev	$L_Z = 0.9 + 6.5 \tan \gamma_s$	$5 \leq T_L \leq 6.5$	$\gamma_s \leq 65^\circ$
LBL	$L_Z = (1.376T_L - 1.81) \tan \gamma_s - 0.38$	$1.5 \leq T_L \leq 8.5$	$2^\circ \leq \gamma_s \leq 76^\circ$

†Except for the last equation, this table is based on Ref. [5].

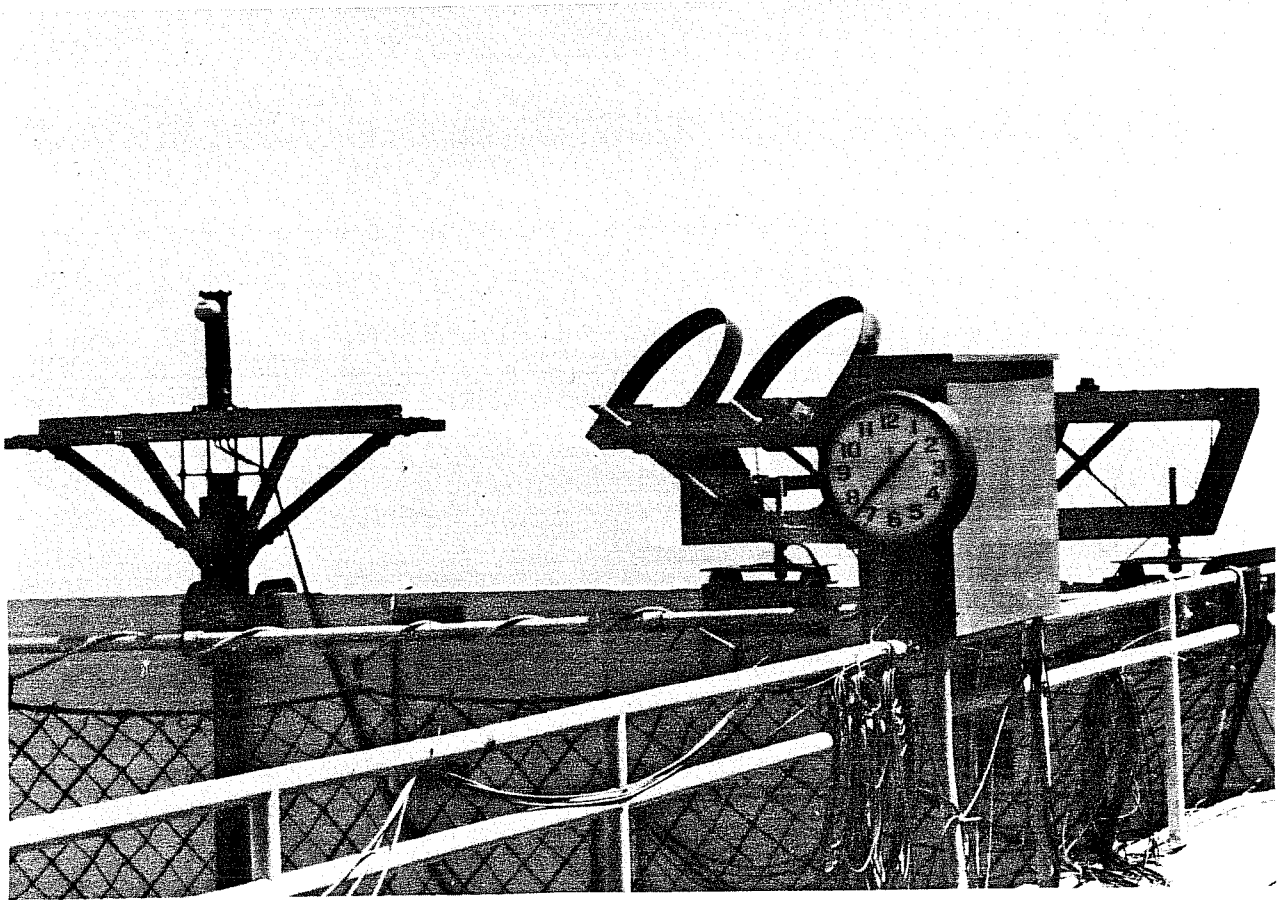
Table 2. Coefficients of Relative Luminance Equations

	Kittler	Gusev
A_0	6.9731	7.6752
A_1	4.2496 E-2	6.1096 E-2
A_2	-8.5375 E-4	-5.9344 E-4
A_3	-8.6088 E-5	-1.6018 E-4
A_4	1.9848 E-6	3.8082 E-6
A_5	-1.6222 E-8	-3.3126 E-8
A_6	4.7823 E-11	1.0343 E-10

Table 3. Parameters of Diffusion Indicatrix Equations

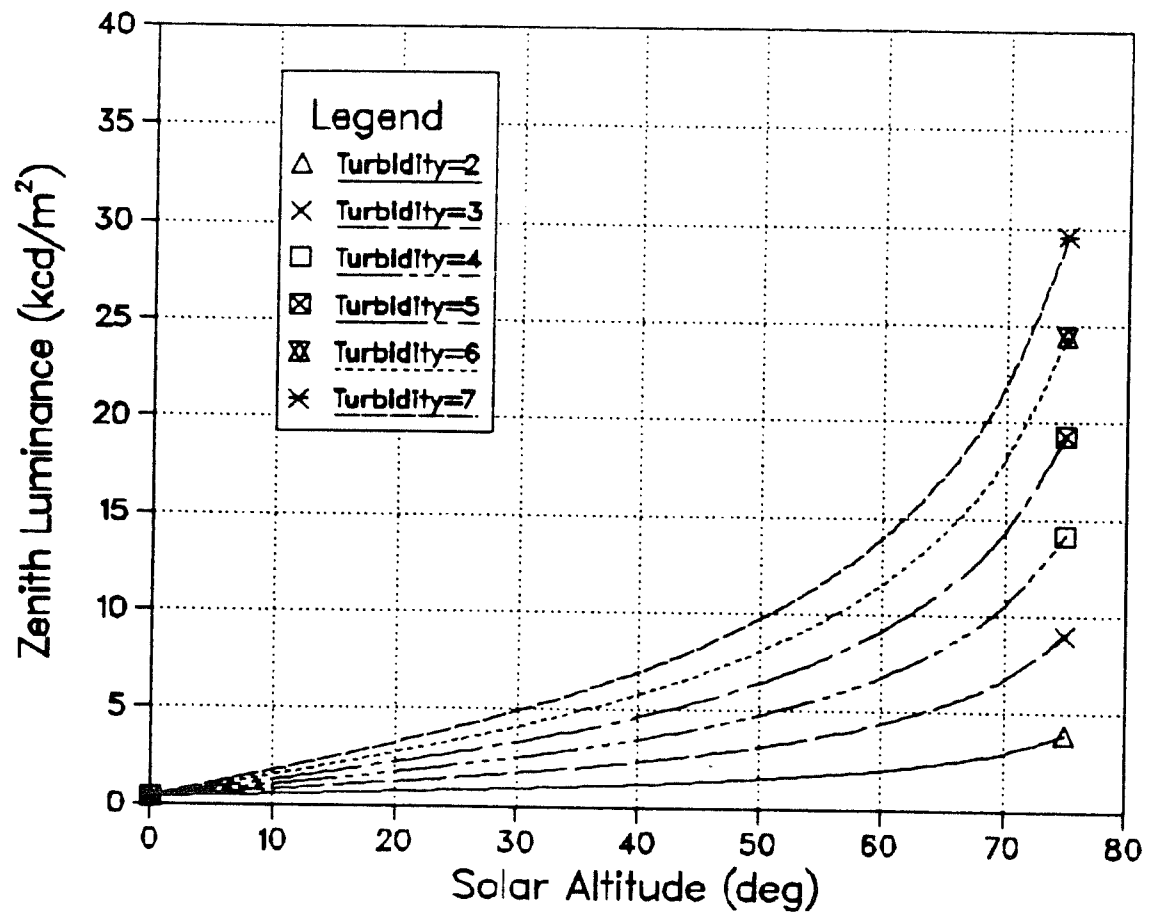
	Kittler	Gusev	LBL
P_1	1	1	1.05 (0.90 - 1.2) [†]
σ_1			0.07
P_2	10 (5 - 20)	16	15 (7 - 29)
σ_2			6
P_3	0.45 (0.3 - 0.8)	0.3	-0.7 (-3.9 - 0.6)
σ_3			1.3
P_4	0.32 (0.265 - 0.32)	0.32	0.4 (0.29 - 0.75)
σ_4			0.1

[†]The values in parenthesis show ranges of parameters, while the value outside the parenthesis are the best estimates.



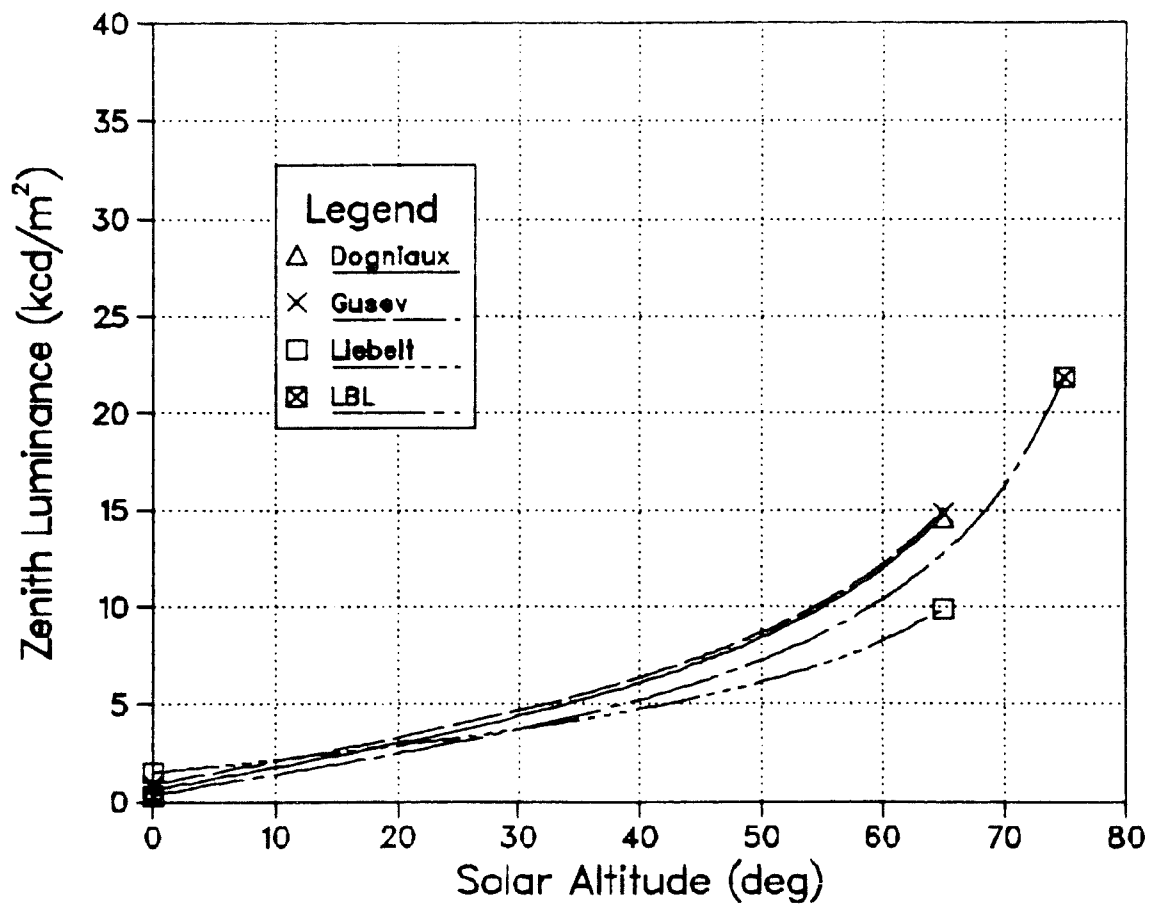
XBB 832-1029

Figure 1. Photometric and radiometric instrumentation on the roof of the Pacific Gas and Electric building in downtown San Francisco.



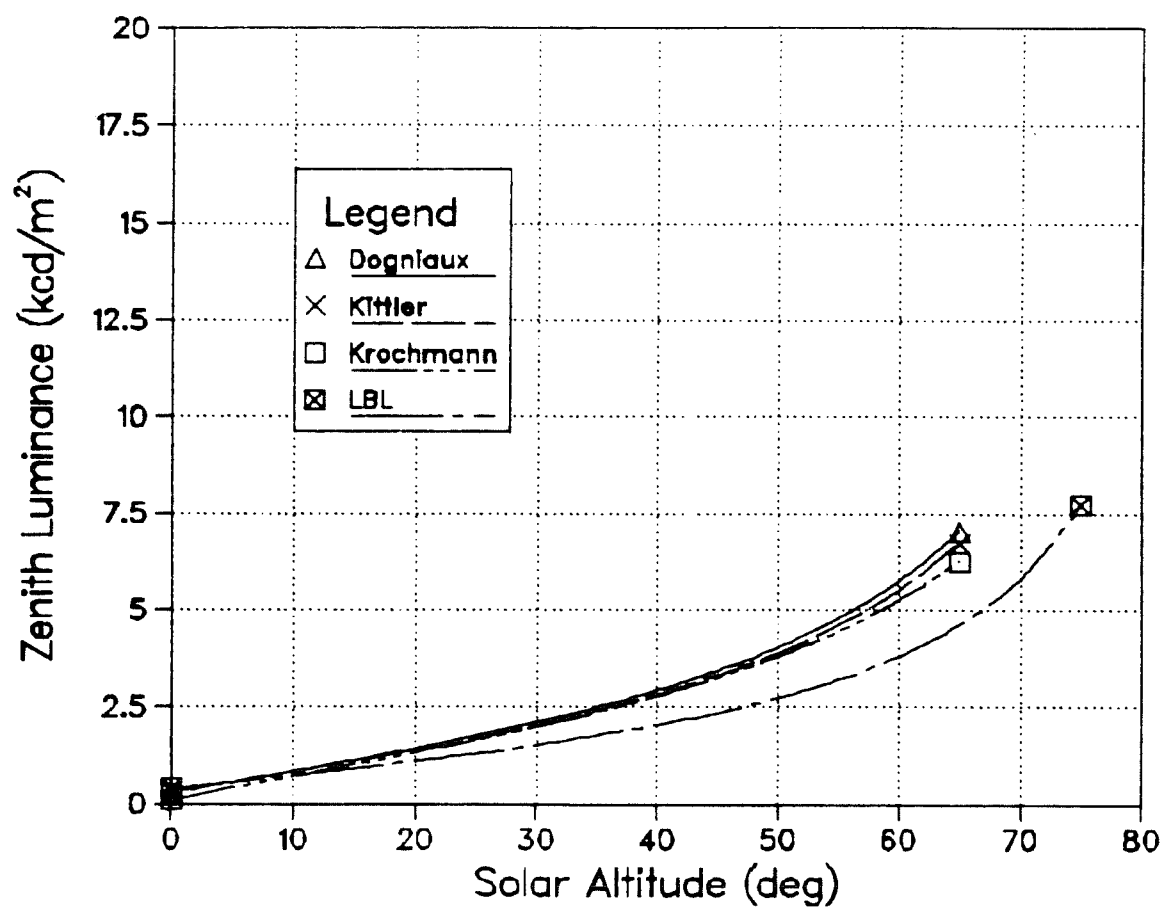
XBL 845-2042

Figure 2. Zenith luminance as a function of solar altitude for varying turbidity.



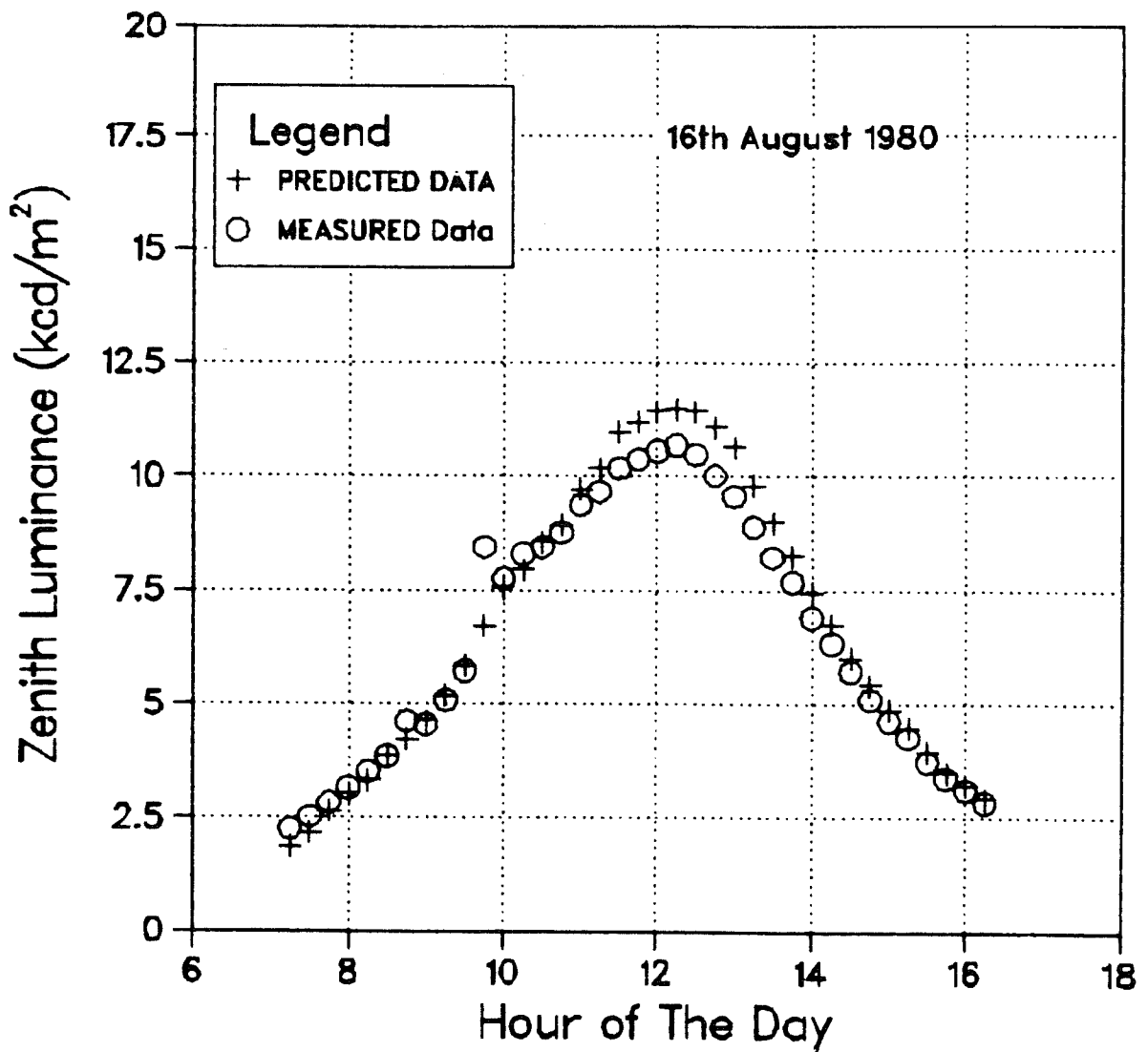
XBL 845-2043

Figure 3. Comparison of zenith luminance equations for high turbidity, $T = 5.5$ (see Table 1 for equations).



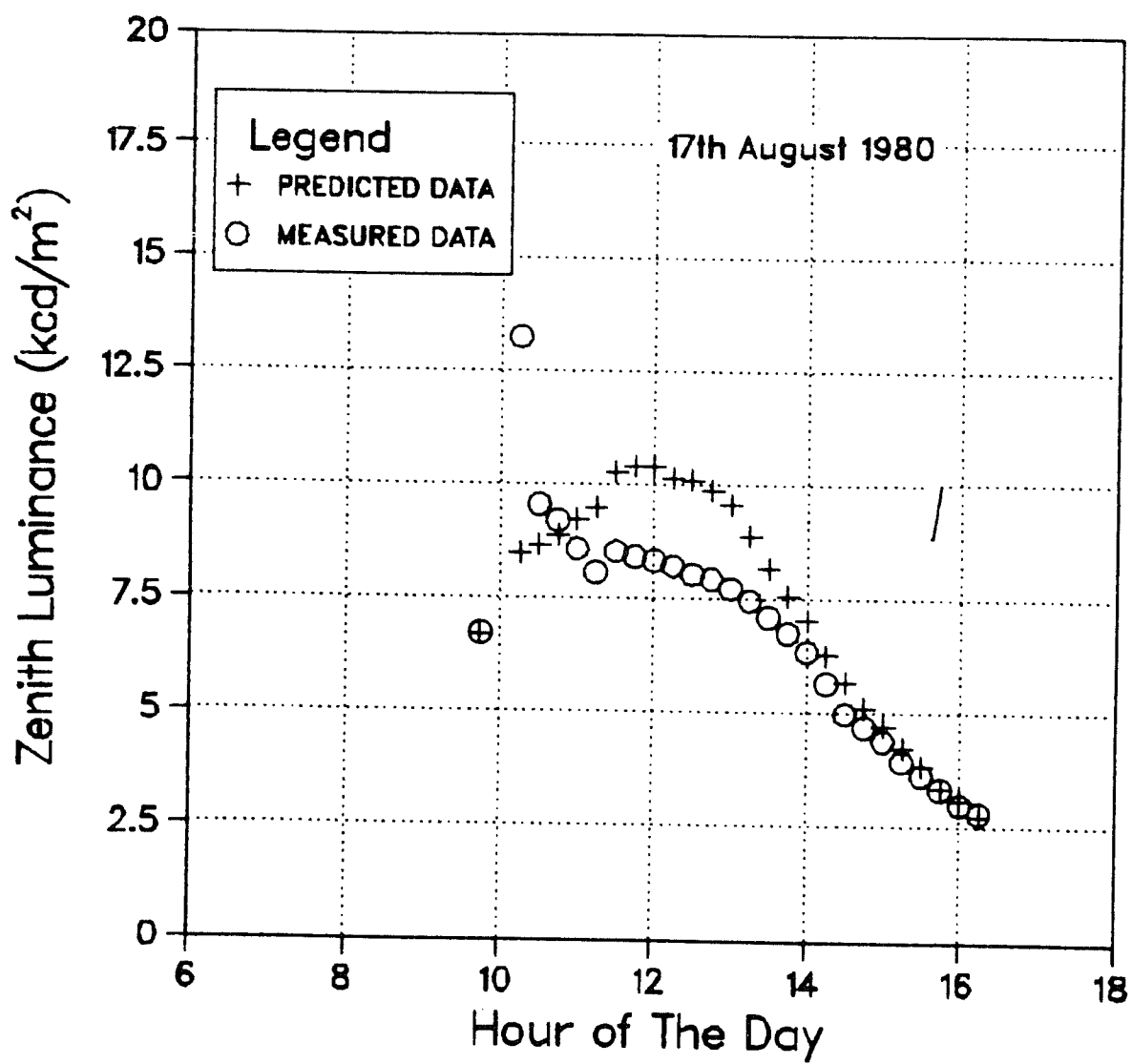
XBL 845-2044

Figure 4. Comparison of zenith luminance equations for low turbidity, $T = 2.75$ (see Table 1 for equations).



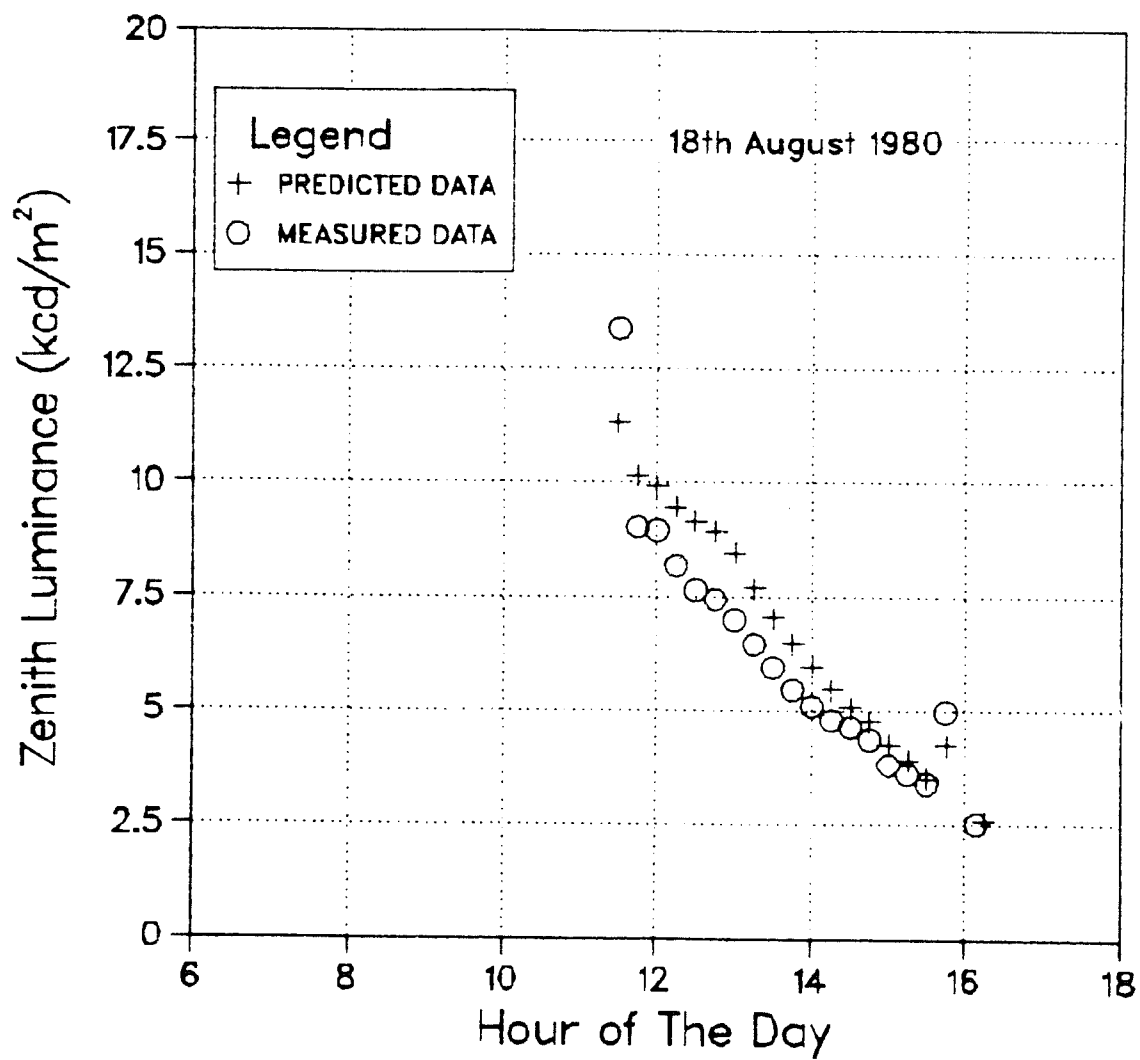
XBL 845-2045

Figure 5a,b,c. Comparison of measured zenith luminance with predicted values using Eq. (1) from three randomly selected consecutive summer days. Hours with missing data did not meet clear day criterion.



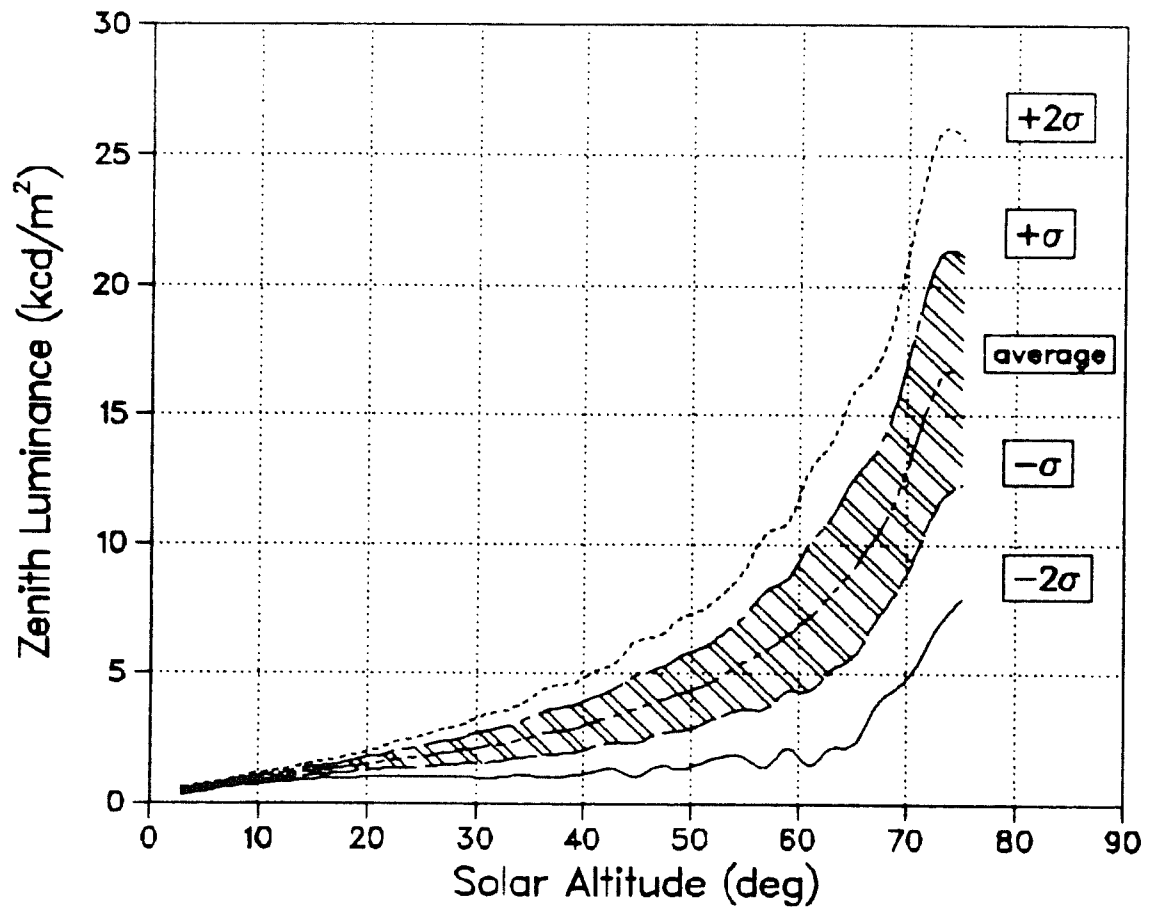
XBL 845-2046

Figure 5 b.



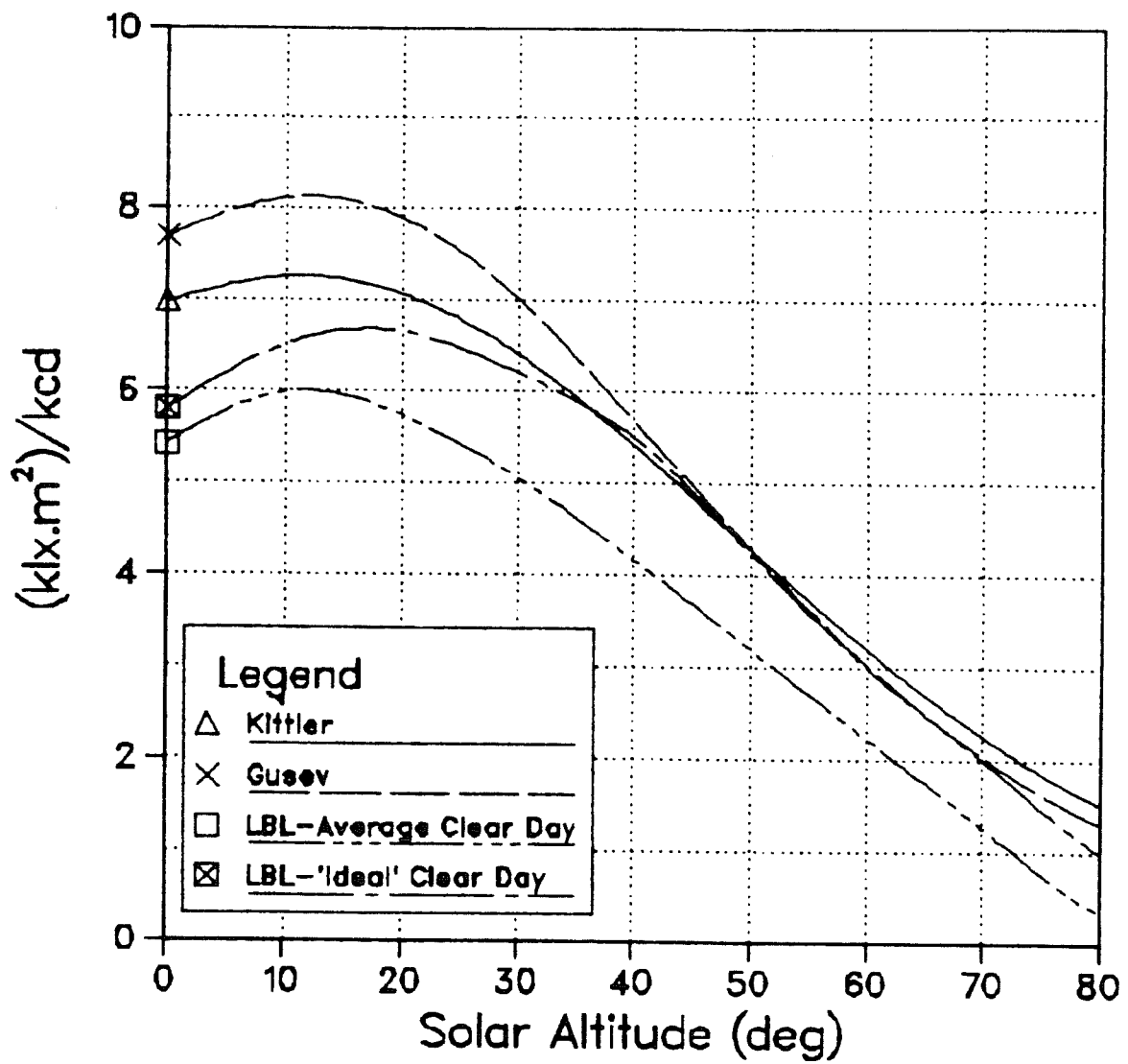
XBL 845-2047

Figure 5. c.



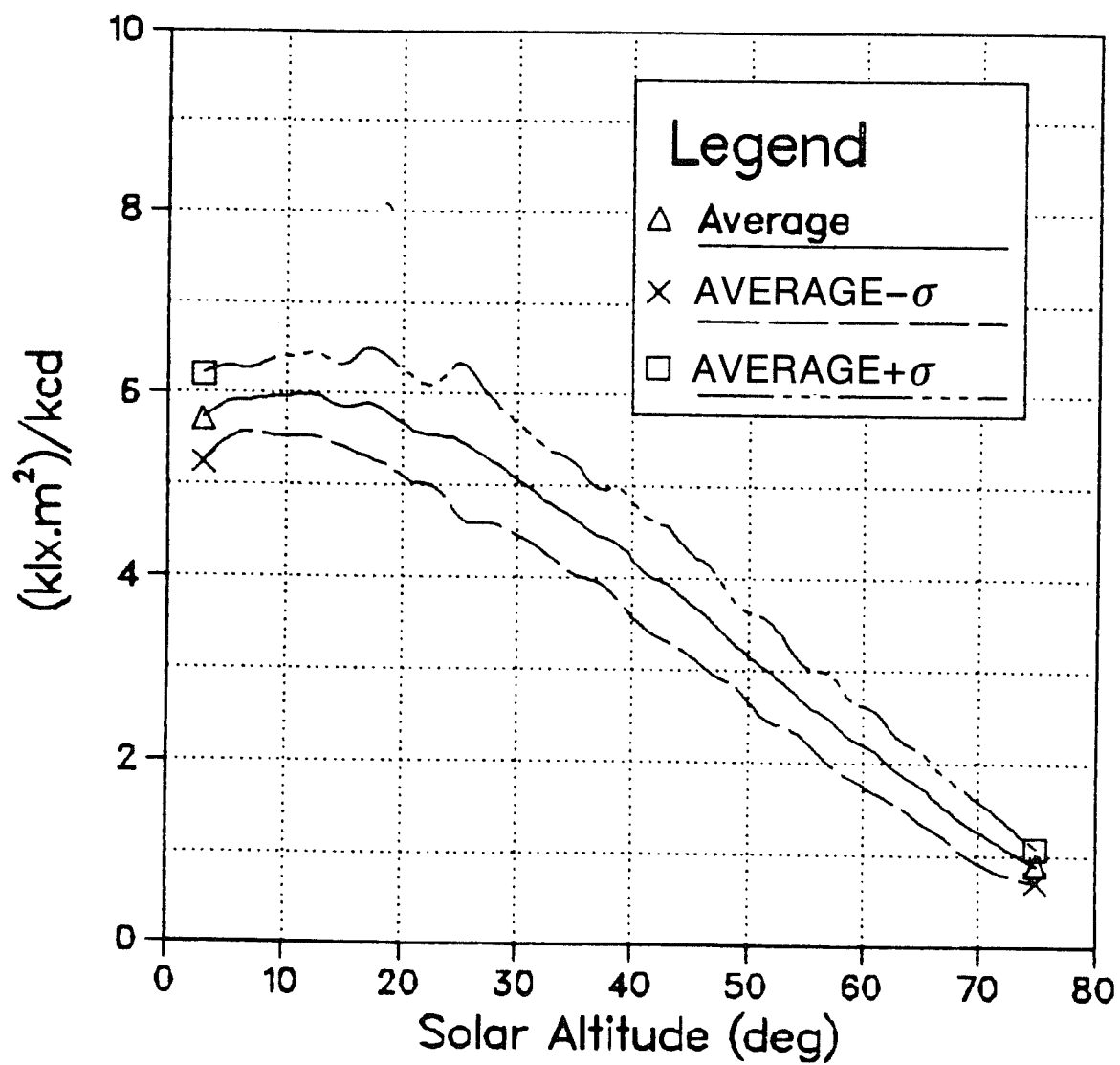
XBL 845-2048

Figure 6. Deviation of zenith luminance measurements (as a function of solar altitude) from overall average value if turbidity is not specified.



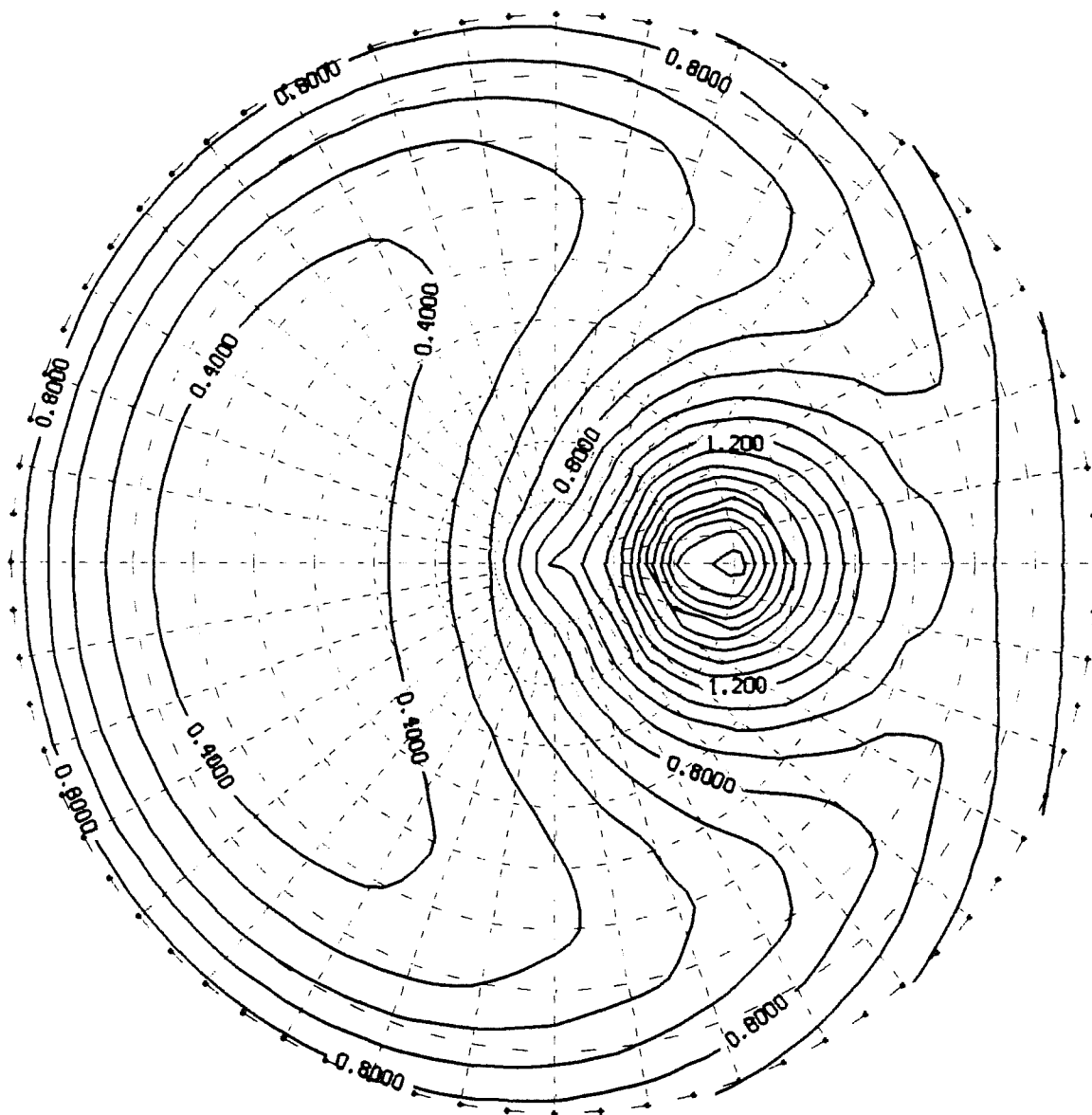
XBL 845-2049

Figure 7. Relative zenith luminance: ratio of diffuse horizontal illuminance to zenith luminance as a function of solar altitude.



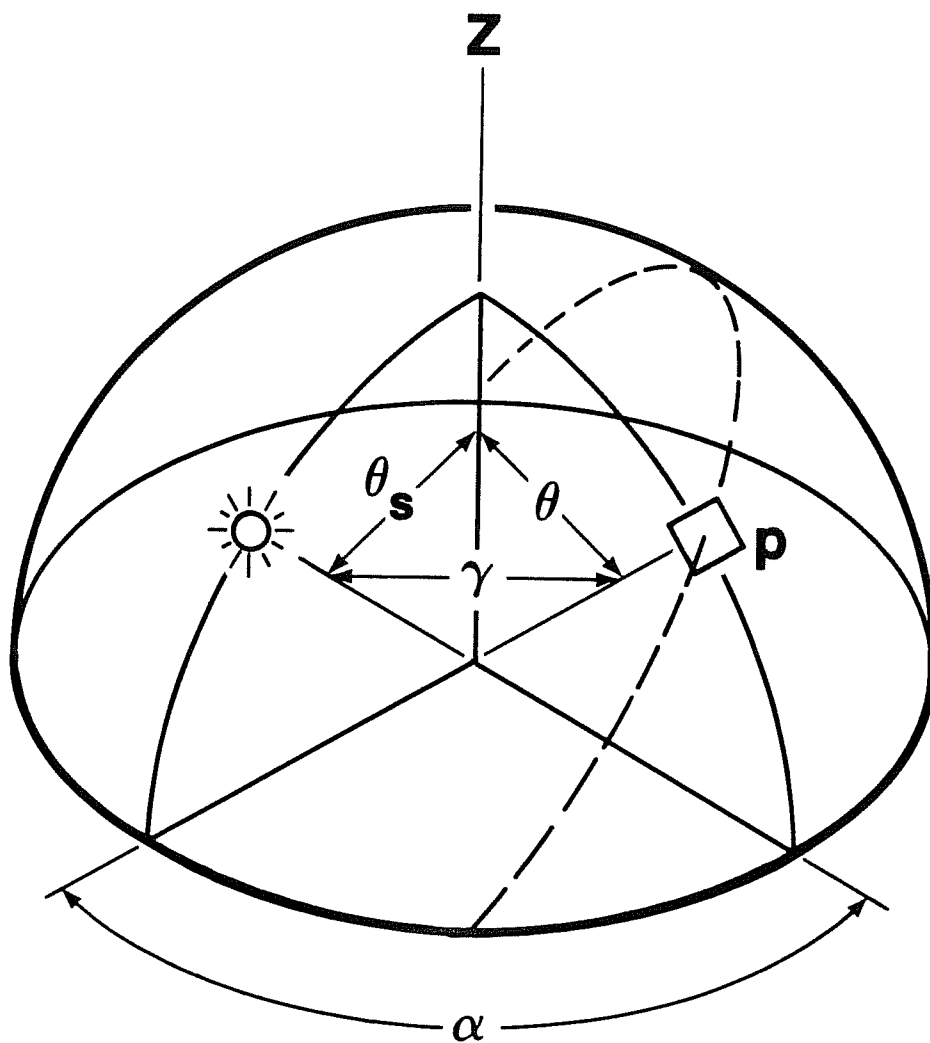
XBL 845-2057

Figure 8. Relative zenith luminance: measured average (\pm one standard deviation) ratio of diffuse illuminance to zenith luminance as a function of solar altitude.



XBL 845-1886

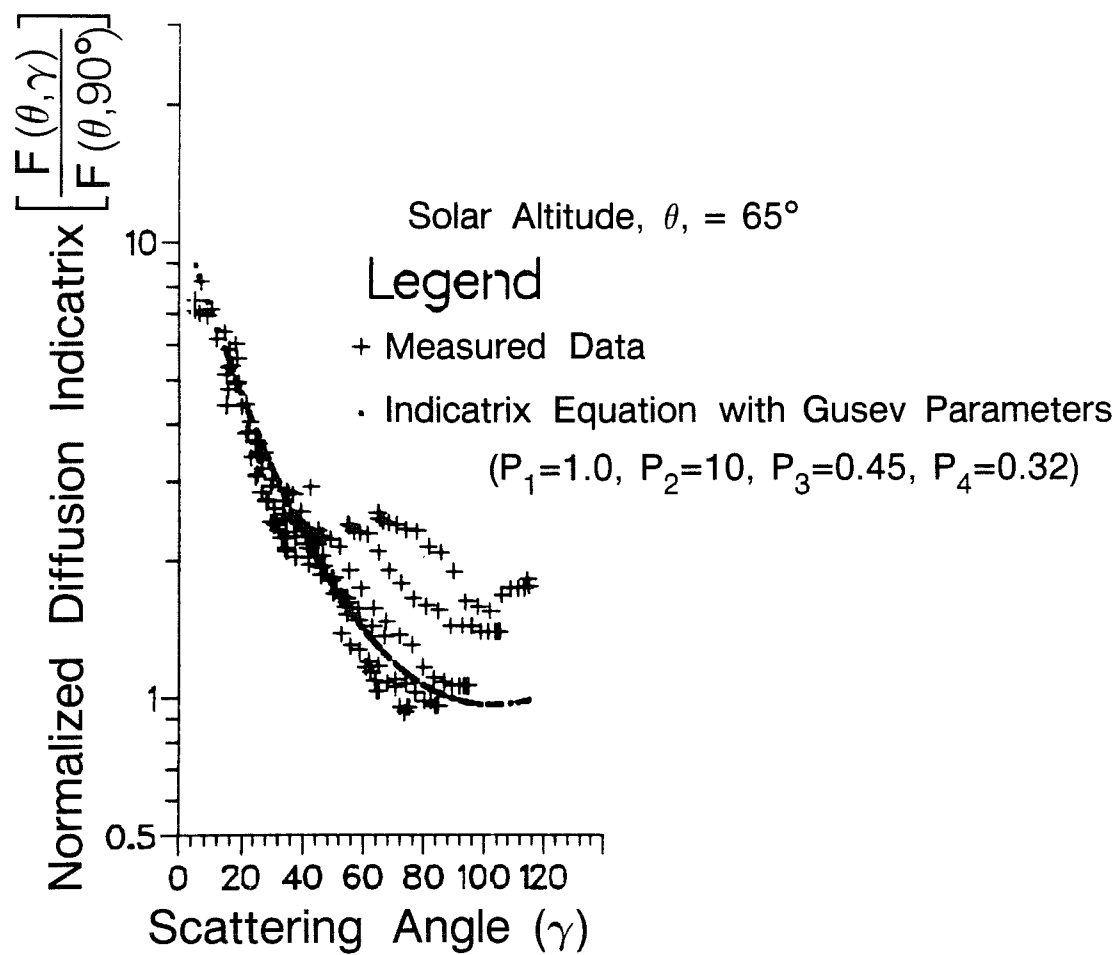
Figure 9. Clear sky luminance distribution based on measured data for sun altitude = 60°.



γ = angle between the sky element, **p**, and the sun,
 θ = angle between the sky element and the zenith,
 θ_s = angle between the sun and the zenith, and
 α = azimuth angle between the sun azimuth and the
 sky element azimuth

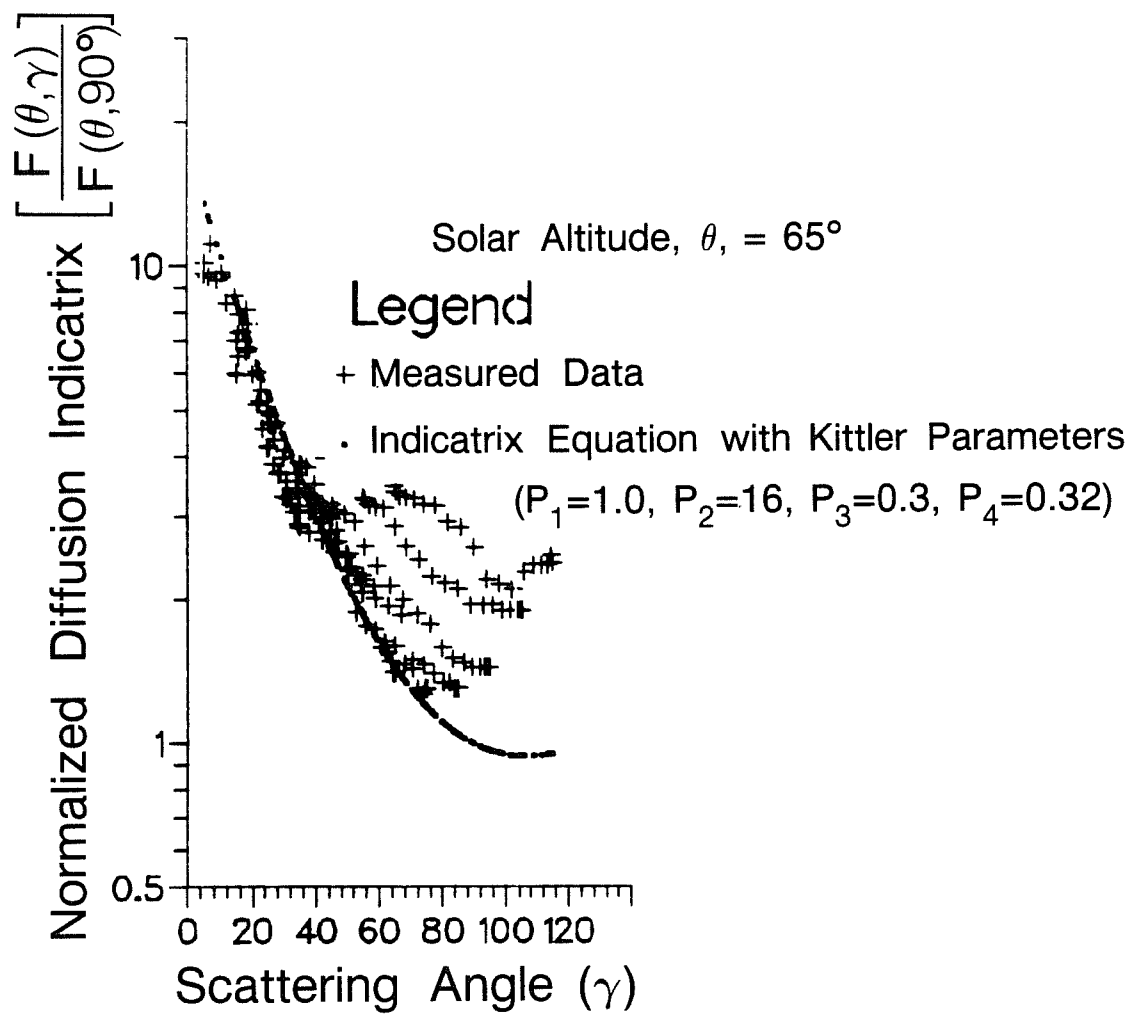
XBL 844-8396

Figure 10. Definition of angles for calculation of the scattering angle γ . The dashed line shows the locus of sky elements having the same γ angle.



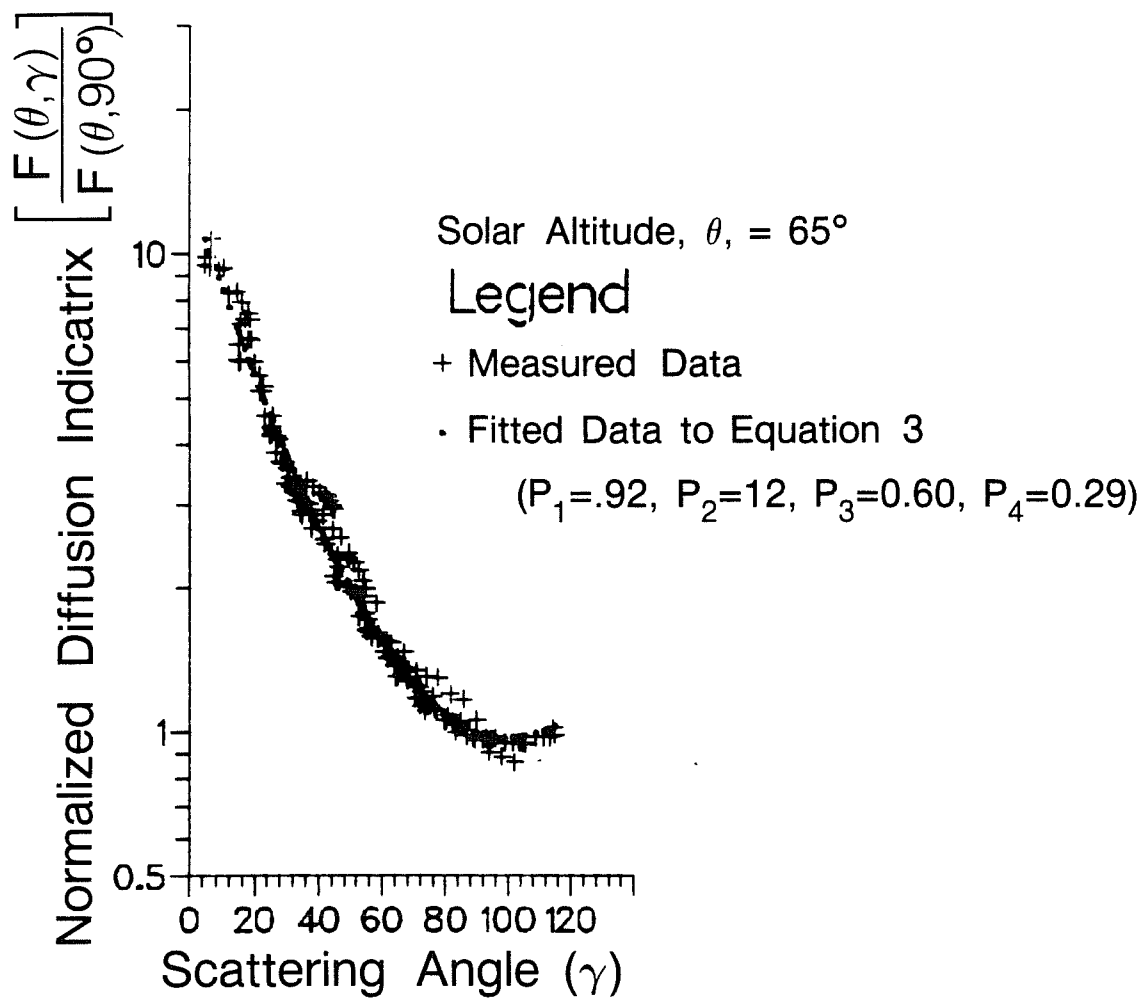
XBL 845-9373

Figure 11a,b,c. Diffusion indicatrix versus scattering angle. Measured data vs. Kittler's equation (11a), Gusev's equation (11b), and LBL equation (11c).



XBL 845-9372

Figure 11 b.



XBL 845-9375

Figure 11 c.